PROPAGATION OF DISTURBANCES IN A LIQUID CONTAINING VAPOR BUBBLES

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The structure and dynamics of waves in a vapor–liquid medium are investigated on the basis of a model equation for wave propagation in a liquid containing vapor bubbles. The results of the calculations are compared with the experimental pressure profiles.

1. A two-temperature model has been proposed [1] for the propagation of disturbances in a liquid existing near the saturation line and containing vapor bubbles. On the assumption that the thermodynamic equilibrium condition at the bubble–liquid interface is preserved in wave transmission, an equation has been derived in [1], describing the one-way propagation of a pressure wave:

$$\frac{\partial p}{\partial t}+c_0\frac{\partial p}{\partial x}+\alpha c_0\frac{p}{p_0}\frac{\partial p}{\partial x}+\beta c_0\frac{\partial^3 p}{\partial x^3}=\frac{3\gamma p_0}{2R\rho_2 L}q_L, \quad (1.1)$$

where $c_0 = (\gamma p_0/\rho_0\ _0)^{1/2}$ is the sound velocity in the vapor–liquid medium; $\beta = R_0^2/6\varphi_0(1-\varphi_0)$, dispersion parameter of the medium; ρ_2, density of the vapor; L, latent heat of vaporization; q_L, heat flux from the bubble into the liquid; α, nonlinearity parameter in the wave.

It has been assumed in the derivation of Eq. (1.1) that the heat flux q_V into the bubble is much smaller than q_L. This assumption is allowable for $\lambda_2 \ll \lambda_1$ and $\sqrt{a_2/a_1} > 1$, where λ_1, λ_2 are the thermal conductivities of the liquid and the vapor and a_1, a_2 are the thermal diffusivities of the liquid and the vapor.

The heat flux q_L in the model of [1] is written in the Duhamel integral form [2]

$$q_L=-\frac{\lambda_1(T-T_s)}{R_0}-\lambda_1\int_0^t\frac{\partial}{\partial\tau}(T-T_s)\frac{1}{\sqrt{\pi a_1(t-\tau)}}d\tau. \quad (1.2)$$

The approximation (1.2) postulates not only weak mobility of the bubble boundary, but also the fact that the thermal wavelength $l_T = \sqrt{2a_1/\omega}$ is much smaller than the distance between the bubbles. When l_T is of the order of the acoustic wavelength l_a, many bubbles fit within the wavelength, and the model of [1] does not work. This situation corresponds to the problem of the sound velocity in the vapor–liquid medium considered in [3] and the wave-propagation model formulated in [4]. In this sense the proposed model of [1] is a high-frequency model.

Assuming that the wave amplitude is small and the compressibility of the vapor can be neglected, we can relate the temperature perturbation ΔT to the pressure perturbation according to the Clausius–Clapeyron equation and rewrite the heat flux (1.2) in terms of the pressure perturbation:

$$q_L=\frac{a_1c_p\rho_l T_{s0}}{L\rho_2}\left(\frac{\Delta p}{R_0}+\int_0^t\frac{\partial p/\partial\tau}{\sqrt{\pi a_1(t-\tau)}}d\tau\right).$$

Then after certain estimates and simplifications [1] Eq. (1.1) acquires the form

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$$\frac{\partial p}{\partial t}+c_0\frac{\partial p}{\partial x}+\frac{\gamma+1}{2\gamma}\frac{\Delta p}{p_0}\frac{\partial p}{\partial x}+\beta c_0\frac{\partial^3 p}{\partial x^3}=-\frac{3\gamma p_0 a_1 c_p \rho_1 T_{s0}}{2R_0^2\rho_2^2 L^2}\left(\Delta p+\frac{R_0}{\sqrt{\pi a_1}}\int_0^t\frac{\partial p/\partial\tau}{\sqrt{t-\tau}}d\tau\right). \tag{1.3}$$

Equation (1.3) is generalized to the case involving dissipation due to the acoustic radiation viscosity by analogy with [5]. The left-hand side of Eq. (1.4) acquires a term of the form $\eta\partial^2 p/\partial x^2$, where η is the effective dissipation coefficient.

With the introduction of the dimensionless variables

$$u=((\gamma+1)/2\gamma)c_0\Delta p/p_0,\qquad u_0=((\gamma+1)/2\gamma)c_0\Delta p_0/p_0,\qquad l=u_0t_0,\ \tilde{u}=u/u_0,\ \tau=t/t_0,\ \xi=x/l$$

(t_0 is a characteristic time) Eq. (1.3) acquires the form

$$\frac{\partial\tilde{u}}{\partial\tau}+\tilde{u}\frac{\partial\tilde{u}}{\partial\xi}+\mathrm{M}^{-1}\frac{\partial\tilde{u}}{\partial\xi}-\frac{1}{\mathrm{Re}}\frac{\partial^2\tilde{u}}{\partial\xi^2}+\frac{1}{\sigma^2}\frac{\partial^3\tilde{u}}{\partial\xi^3}=-\frac{\sigma^2 c_0}{4\mathrm{Pe}\,\mathrm{M}c_2^2}\tilde{u}-\frac{1}{4}\sqrt{\frac{6\varphi_0}{\pi}}\frac{\sigma^2 c_0}{\sqrt{\mathrm{Pe}\,\mathrm{M}c_2^2}}\int_0^\tau\frac{\partial\tilde{u}/\partial\tau^*}{\sqrt{\tau-\tau^*}}d\tau^*, \tag{1.4}$$

where

$$\mathrm{Re}=u_0l/\eta;\quad \sigma^2=l^2u_0/\beta c_0;\quad \mathrm{Pe}=u_0l/a_1;$$
$$\mathrm{M}=u_0/c_0;\quad c_2^2=L^2\rho_2^2/T_{s0}\rho_1^2c_p;$$

c_2 coincides formally with the expression for the sound velocity in the vapor–liquid mixture in [3].

For water containing vapor bubbles at one atmosphere $\mathrm{Pe}\sim 10^8$, so that the term proportional to $\tilde{u}$ can be neglected. The transition to the variables $\theta=\tau\sigma$ and $\zeta=\xi\sigma$ completes the solution of the problem of the principal criteria governing the wave process in the liquid containing vapor bubbles:

$$\frac{\partial\tilde{u}}{\partial\theta}+\mathrm{M}^{-1}\frac{\partial\tilde{u}}{\partial\zeta}+\tilde{u}\frac{\partial\tilde{u}}{\partial\zeta}-\frac{\sigma}{\mathrm{Re}}\frac{\partial^2\tilde{u}}{\partial\zeta^2}+\frac{\partial^3\tilde{u}}{\partial\zeta^3}=-W\int_0^\theta\frac{\partial\tilde{u}/\partial\theta^*}{\sqrt{\theta-\theta^*}}d\theta^*;$$
$$W=\frac{\gamma+1}{2\gamma}\sqrt{\frac{3}{8\pi\varphi_0}}\,\mathrm{Ja}\sqrt{\frac{\sigma}{\mathrm{Pe}}\mathrm{M}^3}, \tag{1.5}$$

where $\mathrm{Ja}=c_p\Delta T\rho_1/L\rho_2$ is the Jakob number.

As $W\to 0$ the propagation of waves in the liquid containing bubbles is determined, as in gas–liquid systems, by the values of σ and Re, and the involvement of phase transitions in the wave is characterized by the criterion W. The latter varies as a function of the initial pressure p_0 and the physical parameters of the wave: Pe.

2. Equations (1.4) and (1.5) are the Burgers–Korteweg–de Vries (BKdV) relaxation equations. The right-hand sides of these equations contain a relaxation integral. Unlike the BKdV relaxation equation derived in [6] for the modeling of waves in a liquid containing gas bubbles with heat transfer, the integral has a "square root" kernel, rather than an exponential kernel as in the case of [6]. With an exponential kernel it is possible to determine explicitly the characteristic relaxation time τ_0 and, by differentiating, to eliminate the integral, arriving at a higher-order equation. The "square root" kernel corresponds to an infinite relaxation time and does not permit the transition to a higher-order differential equation without an integral.

The propagation of waves in a liquid containing vapor bubbles is modeled on the basis of Eqs. (1.4) and (1.5) with the application of numerical integration to the experiments of [7]. Equation (1.4) is integrated numerically for $\mathrm{Re}\to\infty$ according to an asymmetric difference scheme [8]:

$$\tilde{u}_{i+3}^{n+1}=\tilde{u}_i^{n-1}-\frac{\Delta\tau}{\Delta\xi}\left((\tilde{u}_{i+1}^n)^2-(\tilde{u}_{i+2}^n)^2\right)-\frac{2\Delta\tau}{\Delta\xi^3\sigma^2}\left(\tilde{u}_{i+3}^n-3\tilde{u}_{i+2}^n+3\tilde{u}_{i+1}^n-\tilde{u}_i^n\right)-\varepsilon I_{i+1.5}^n,$$

where ε is the coefficient in front of the integral in Eq. (1.4) and $I_{i+1.5}^n$ is an approximation of the integral, written in the following form for the net-point (computing grid) representation of the function $\tilde{u}(\tau,\xi)$

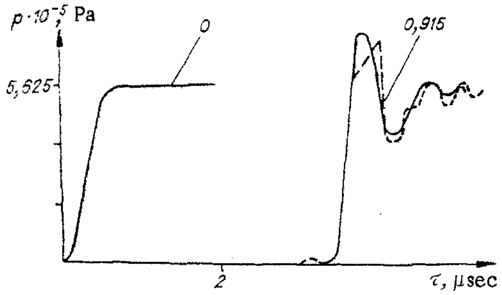

Fig. 1

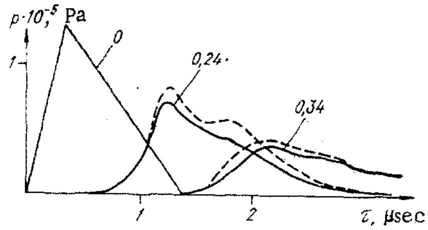

Fig. 2

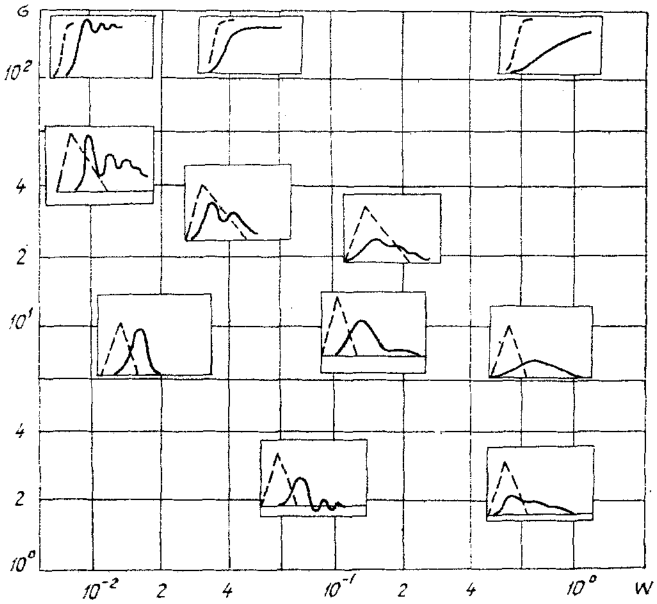

Fig. 3

$$I^{n}_{i+1.5} = \frac{2}{\sqrt{\Delta\tau}}\sum_{k=1}^{n-1}\left(\tilde{u}^{k+1}_{i+1.5} - \tilde{u}^{k}_{i+1.5}\right)\left(\sqrt{n-k} - \sqrt{n-1-k}\right), \qquad (2.2)$$

where $\eta = \tau/\Delta\tau$.

In Eq. (2.1) $\Delta\xi$ and $\Delta\tau$ are related by the stability condition $\Delta\tau \leqslant \Delta\xi^3\sigma^2/8$ and the approximation condition $1.5\Delta\xi/\Delta\tau = M^{-1}$.

The scheme (2.1) is implemented as a τ-explicit scheme, making it possible to compute the values of the function $u(\tau, \xi)$ directly on a four-word array with respect to ξ.

The numerical solutions of Eq. (1.5) are found by an analogous procedure. The operation of the scheme is verified in three stages. In the first stage we set $\varepsilon = 0$, whereupon Eq. (1.5) goes over to the Korteweg–de Vries equation, which has well-known numerical solutions [9]. In the second stage, to verify expression (2.2) we compare the numerical solution of the problem

$$\frac{\partial u}{\partial t} + M^{-1}\frac{\partial u}{\partial x} = -\varepsilon\int_0^t \frac{\partial u/\partial\tau}{\sqrt{t-\tau}}\,d\tau, \quad u(t, 0) = \begin{cases} 0 & \text{for } t = 0, \\ 1 & \text{for } t > 0 \end{cases}$$

with its analytical solution

$$u(t, x) = \operatorname{erfc}\left(\frac{\varepsilon\sqrt{\pi}Mx}{2\sqrt{t - Mx}}\right).$$

In addition, we compare the numerical solution of the linearized equation (1.5) with the solution obtained by the fast Fourier transform method from the derived dispersion relation [8]. In every case the error does not exceed 2%.

The values of the coefficients of Eq. (1.5) are calculated from the initial conditions of the experiments [7, 9], and the equations are solved at distances X_i corresponding to the coordinates of the sensors.

3. The results of the calculations are compared with the experimental pressure profiles. Figure 1 shows the results of calculations of a disturbance of the "shock wave" type; here and in the other figures the dashed curves represent the experimental results ($\sigma \to \infty$, $W = 62 \cdot 10^{-4}$, $M = 0.67$).

Figure 2 shows the results of the calculations for the structure of a wave of finite extent and compares them with the experimental results ($\sigma = 26.5$, $W = 0.67$, $M = 0.67$).

The results of the calculations are conveniently represented in the form of a graphical tableau, the coordinates of which are the characteristic parameters W, σ of the wave process in a liquid containing vapor bubbles (Fig. 3); this graphical representation is similar to [7].

The numerical modeling of Eqs. (1.4) and (1.5) and the comparison of the results of the calculations with the experimental data show that the propagation of low-intensity waves in a liquid containing vapor bubbles is adequately described by the Burgers—Korteweg—de Vries equation with a "square root" kernel.

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